“4/3 Problem” Resolution

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http://www.softcom.net/users/der555/elecmass.pdf

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I would like to offer a resolution to the famous “4/3 problem” of electrodynamics. The theory of relativity implies that the momentum of the field of an electron must be the same as the rest energy of the field times v/c^2, where v is the magnitude of the velocity of the electron. However, the momentum of the field, calculated from the Poynting vector, is 4/3 times the energy of the field times v/c^2. Until now, this “4/3 problem” has gone unresolved (this non-relativistic, three-dimensional treatment is derived from my relativistic, four-dimensional equations at http://www.softcom.net/users/der555/newtransform.pdf).

I will show, here, that the mass of an electron is *entirely* 'electromagnetic' in origin. I'll be using a derivation which closely parallels the one that Richard Feynman uses in "Lectures on Physics", vol. 2, Sections 28-1 through 28-3. I've included, however, the necessary additions that Feynman didn't consider. In order to conform with Feynman's derivation, I've used SI units, here, in contrast to the Gaussian units used in the paper, above.

The value that the mass m_{elec}, derived from the momentum of the field, must have to be considered entirely 'electromagnetic' in origin is the energy of the field, U_{elec}, divided by c^2, or

$$m_{\mathrm{elec}} = \frac{U_{\mathrm{elec}}}{c^2} \tag{1}$$

The value for U_{elec}, which Feynman calculated in Section 28-1, Eq. (28.2), is

$$U_{\mathrm{elec}} = \frac{1}{2}\frac{e^2}{a} \tag{2}$$

where a is the lower limit of integration of the field energy density, and

$$e^2 = \frac{q^2}{4\pi\epsilon_0} \tag{3}$$

where q is the charge of the electron and ϵ_0 is the permittivity constant. However, as I have shown elsewhere,[1] the factor 1/2 should not appear and the value for U_{elec} should, instead, be

$$U_{\mathrm{elec}} = \frac{e^2}{a} \tag{4}$$

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[1] See http://www.softcom.net/users/der555/enerdens.pdf

So in order to be considered entirely electromagnetic, in origin, our m_{elec} needs to be

$$m_{\text{elec}} = \frac{e^2}{ac^2} \tag{5}$$

Suppose that the electron is in uniform motion with velocity $v \ll c$. The momentum density of the field $\mathbf{g} = \epsilon_0 \mathbf{E} \times \mathbf{B}$, where $\mathbf{E}$ and $\mathbf{B}$ are the conventional electric and magnetic field three-vectors, is directed obliquely to the line of motion for an arbitrary point P at a distance r from the center of the charge.[2] The magnetic field is $\mathbf{B} = \mathbf{v} \times \mathbf{E}/c^2$, which has the magnitude $(v/c^2)E \sin\theta$, where θ is the angle between $\mathbf{v}$ and $\mathbf{E}$. The momentum density $\mathbf{g}$, then, has the magnitude

$$g = \frac{\epsilon_0 v}{c^2}\, E^2 \sin\theta \tag{6}$$

The fields are symmetric about the line of motion, so when we integrate over space, the transverse components will sum to zero, giving a resultant momentum parallel to $\mathbf{v}$. The component of $\mathbf{g}$ in this direction is $g \sin\theta$ or, from (6)

$$g \sin\theta = \frac{\epsilon_0 v}{c^2}\, E^2 \sin^2\theta \tag{7}$$

However, the momentum due to $g \sin\theta$ alone, when integrated over all space and divided by v, as Feynman points out later, gives a value for m_{elec} of

$$m_{\text{elec}} = \frac{4}{3}\,\frac{U_{\text{elec}}}{c^2} \tag{8}$$

which is, clearly, not the same as the value from (1) required in order for the mass of the electron to be entirely electromagnetic in origin. This is the “4/3 problem” problem.[3]

I would like to consider, now, a contribution to the momentum density from $\mathbf{h} = -\epsilon_0 \mathbf{E}(\nabla \cdot \mathbf{A})$, where $\mathbf{A}$ is the vector potential.[4] Since $\mathbf{A} = \mathbf{v}\phi/c^2$, where ϕ is the static electric potential, we can also write $\nabla \cdot \mathbf{A}$ as

$$\nabla \cdot \mathbf{A} = \nabla \cdot \left(\frac{\mathbf{v}\phi}{c^2}\right) = \frac{1}{c^2}\,\mathbf{v} \cdot (\nabla\phi) = -\frac{1}{c^2}\,\mathbf{v} \cdot \mathbf{E} \tag{9}$$

The magnitude of $\mathbf{v} \cdot \mathbf{E}/c^2$ is $(v/c^2)E \cos\theta$, so the magnitude of $\mathbf{h}$ is

$$h = \frac{\epsilon_0 v}{c^2}\, E^2 \cos\theta \tag{10}$$

The component of $\mathbf{h}$ in the direction of $\mathbf{v}$ is $h \cos\theta$ or, from (10)

$$h \cos\theta = \frac{\epsilon_0 v}{c^2}\, E^2 \cos^2\theta \tag{11}$$

So the total momentum density is given by $g \sin\theta + h \cos\theta$. We now have to integrate the total momentum density over all space to find the total field momentum p.[5] Feynman takes the volume element as $2\pi r^2 \sin\theta \, d\theta \, dr$, so that the total momentum is

$$p = \int_{\text{all space}} (g \sin\theta + h \cos\theta)\, 2\pi r^2 \sin\theta \, d\theta \, dr \tag{12}$$

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[2] Refer to Fig. 28-1, page II-28-2 in Feynman’s ”Lectures on Physics”.

[3] Actually, since the factor 1/2 in (2) has been removed, it should be called the ”2/3 problem”.

[4] This is not an *ad hoc* addition. It is derived from the time component of my electric field four-vector, and is part of the momentum density components of my energy-momentum tensor in the paper at the URL above.

[5] Refer to Fig. 28-2, page II-28-2.

or, using (7) and (11),

$$p = \int_{\text{all space}} \frac{\epsilon_0 v}{c^2}\, E^2 \left(\sin^2\theta + \cos^2\theta\right) 2\pi r^2 \sin\theta \, d\theta \, dr \tag{13}$$

Since $\sin^2\theta + \cos^2\theta = 1$, this reduces to

$$p = \int_{\text{all space}} \frac{\epsilon_0 v}{c^2}\, E^2 \, 2\pi r^2 \sin\theta \, d\theta \, dr \tag{14}$$

The result of integrating this over all space, with the limits of θ being 0 and π, and the limits of r being a and ∞, is

$$p = \frac{e^2}{ac^2}\, v \tag{15}$$

Since $p = mv$, the mass of the electron m_{elec} is

$$m_{\text{elec}} = \frac{e^2}{ac^2} \tag{16}$$

As you can see, this is *exactly* the same value as we got in (5). The value for the mass of the field derived from the energy of the field divided by c^2, and the value for the 'electromagnetic' mass derived from the momentum of the field are *identical*, meaning that the mass of the electron is *entirely* 'electromagnetic' in origin.